SARDAR PATEL INS. OF SCI &TECH. MAHAVIDHYALAY, GORAKHPUR

Pre University Examination-2019 - 2020

B.Sc –I

Time: 3 hours **Subject : Mathematics** **M. M: - 50**

Paper – I (Algebra and trigonometry)

Note: (i) Attempt five questions in all.
(ii) Question No.1 is compulsory.
(iii) Select two questions from each section.
(iv) All question carry equal marks.

Q - 1 Answer all parts of the following.

(a) Prove that:

$$32\cos^6\theta = \cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10$$

(b) Find the condition under which the sum of two roots of the equation

$x^2 - px^2 + qx - r = 0$ is zero

(c) State and prove Euler's theorem.

(d) For any two element a and b of a group, Prove that

$(a\,b)^{-1} = b^{-1}a^{-1}$.

(e) Prove that the relation of congruence modulo m is an equivalence relation.

SECTION- A

Q-2 (a) State and prove division algorithm theorem.

(b) State and prove Fundamental Theorem of arithmetic.

Q-3 **(a)** Solve by Cardon's Method
$x^3 - 15x^2 - 33x + 847 = 0$

(b) Reduce the cubic $x^3 + 6x^2 + 9x + 4 = 0$ to the standard form $x^3 + 3Hx + G = 0$

Or

If $\alpha\ \beta\ \gamma$ are the root of the equation $x^3 + 3qx + r = 0$ then form the equation whose root are $(\beta - \gamma)^2, (\gamma - \alpha)^2, (\alpha - \beta)^2$.

Q-4 (a) Define normal subgroup prove that if H be a subgroup of a group G and N be a normal Subgroup of G, then $H \cap N$ is the normal subgroup of H

(b) Of the $\lfloor n$ permutations on n symbols, prove that $\lfloor n /2$ are even and $\lfloor n /2$ are odd.

Q–5 (a) Define Homomorphism, Kernel of Homomorphism and Isomorphism of groups. State and prove the fundamental theorem of Homomorphism.

(b) Prove that the root of the equation.

$(x-1)^n = x^n$ are $1/2\{1 + i\cot(r\pi)/n\}$, where $r = 0,1,2,3,\ldots\ldots\ldots\ldots, n-1$. and n appositive integer.

SECTION- (B)

Q-6 (a) Define abelian group and prove that the set of n n^{th} roots of unity forms multiplicative Abelian group of order n

(b) define subgroup prove that a non empty subset H of group G is a subgroup of G if $\forall$ a, b $\in$ H => $ab^{-1} \in$ H.

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Q- 7 (a) If $\tan(\theta+i\phi) = \text{cas}\ \alpha + i\sin\alpha$, Prove that

$$\phi = \tfrac{1}{2} \log \tan(\pi/4 + \alpha/2)$$

(b) If $\sin^{-1}(x+iy) = A+iB$, prove that $x^2/(\cosh^2 B) + y^2/(\sinh^2 B) = 1$

Q- 8 (a) Sum the series:

$$\frac{1}{\cos\theta - \cos3\theta} + \frac{1}{\cos\theta - \cos5\theta} + \frac{1}{\cos\theta - \cos7\theta} + \ldots\ldots\ldots \text{ to n terms.}$$

(b) If pand q are the integers relatively prime to each other, Prove that $(\cos\theta + i\sin\theta)^{p/q}$ has exactly q distinct values which can be arranged in G.P

Q-9 (a) Prove that $ah\sec\theta - bk\operatorname{cosec}\theta = a^2 - b^2$ has four roots and that the sum of the four values of θ which satisfy it is equal to an odd multiple of π radians.

(b) Sum the series:

$\operatorname{cosec}\alpha + \operatorname{cosec}\alpha/2 + \operatorname{cosec}\alpha/2^2 +$ -------------- to n terms.

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