

SARDAR PATEL INS. OF SCI & TECH. MAHAVIDHYALAY, GORAKHPUR
Pre University Examination-2019 - 2020

B.Sc -I

Time: 3 hours

Subject : Mathematics

M. M: - 50

Paper - III (Differential equation and Laplace Transforms)

Note: (i) Attempt five questions in all.

(ii) Question No.1 is compulsory.

(iii) Select two questions from each section.

(iv) All question carry equal marks.

Q - 1 (a) Define order and degree of a differential equation. Also find the order and degree of the following differential equation.

$$x^2 (d^3y / dx^3)^4 + (d^2y / dx^2)^7 + \{ x^2 + (dy / dx)^4 \}^2 = 0$$

(b) What is the normal form of the second order linear differential equation?

(c) Solve: $dx / 1 = dy / 3 = dz / \{5z + \tan (y-3x)\}$

(d) Solve Bernoulli's equation $dy/dx + Py = Qy^n$.

(e) Evaluate $L^{-1} [(2s + 1) / s (s + 1)]$

SECTION- A

Q-2 Define homogeneous linear differential equation of higher order.

Prove that: $x^n (d^n y / dx^n) = D (D-1) (D-2) \dots (D - n + 1) y$

And solve $x^2 (d^2y/dx^2) + 4x (dy/dx) + 2y = e^x$

Q-3 (a) Solve $d^2y/dx^2 - (1+4e^x) dy/dx + 3e^{2x}y = e^{2(x+e^x)}$

(b) By Transferring to normal form and removing first derivative, solve the differential equation

$$x^2 (d^2y/dx^2) - 2x dy/dx + (2 + x^2)y = x^3 e^x$$

Q -4 Use laplace transform solve the differential equation

$$d^2y/dx^2 + 9y = \cos 2t, \text{ if } y(0) = 1, y'(0) = -1$$

Q-5 (a) Solve: $(D^2 - 6D + 13)y = 8e^{3x} \sin x$.

(b) Solve: $x^2 (d^2y/dx^2) - (x^2 + 2x)dy/dx + (x + 2)y = x^3 e^x$

SECTION- (B)

Q-6 (a) Solve: $y - x(dy/dx) = a \{y^2 + (dy/dx)\}$

(b) Show that the system of confocal conics $x^2/(a^2 + \lambda) + y^2/(b^2 + \lambda) = 1$, Where a,b, are constant and λ the parameter, is self orthogonal.

Q- 7 (a) Find the inverse laplace transforms of $(2s^2 - 1) / (s^2 + 1)(s^2 + 4)$ and $2s / (s^2 + 4a^2)^2$

(b) Solve: $(D^3 + 2D^2 + D)y = e^{-3x} + x^2 + e^x \sin x$.

Q- 8 (a) apply the method of variation of parameters, to solve $(x+2)d^2y/dx^2 - (2x+5)dy/dx + 2y = (1+x)e^x$

(b) Solve: $dy/dx + x \sin 2y = x^3 \cos^2 y$

Q-9 (a) Find the general and singular solution of the differential equation: $(y-px)^2(1+p^2) = a^2 p^2$

(b) Use laplace transform to evaluation:

$$\int_0^{\infty} t^2 e^{-t} \sin t \, dt$$