SARDAR PATEL INS. OF SCI &TECH. MAHAVIDHYALAY, GORAKHPUR

Pre University Examination-2019 - 2020

B.Sc –I

Time: 3 hours **Subject : Mathematics** **M. M: - 50**

Paper – II (Advanced calculus)

Note: (i) Attempt five questions in all.
(ii) Question No.1 is compulsory.
(iii) Select two questions from each section.
(iv) All question carry equal marks.

Q - 1 (a) State and prove **Euler's Theorem** for homogeneous function.

(b) Prove that $\lceil n+1 = n\lceil n$

(c) Show that the limit of the function

$f(x,y) = 2xy/(x^2+y^2)$

does not exist when $(x,y) \longrightarrow (0,0)$.

(d) If $(x,y) = x^2y^2 - 5x^2 - 8xy - 5y^2$, find the maximum/minimum value of $f(x,y)$.

(e) Expand the function $x^2y + 3y - 2$ inpowers of $(x-1)$ and $(y+2)$.

SECTION- A

Q-2 Stat and prove the **Taylor's Theorem** for the expansion of function of two variables. Also,

obtain, **Taylor's Expansion** of $\tan^{-1}y/x$ about (1,1) up to and including the second degree terms.

Hence compute $f(1.1, 0.9)$

Q-3 (a) State and prove **Dirichlet's Theorem** for three variables. Using it, evaluate the integral

$$\int\int\int x^{l-1}y^{m-1}z^{n-1}dxdydz,$$

Wher x,y,z are always positive but limited by the condition $(x/a)^p+(y/b)^q+(z/c)^r \leq 1$

Q –4 Find the volume of greatest rectangular parallelopiped inscribed in the ellipsoid

$$x^2/a^2+y^2/b^2+z^2/c^2 = 1.$$

Q-5 (a) Evaluate by changing the order of integration:

$$\int_0^1 \int_0^{\sqrt{2-x^2}} xdxdy/\sqrt{x^2+y^2}$$

(b) Show that

$$\int_0^\infty x^c/c^x dx = \lceil c+1 \,/\, (\log c)^{c+1}$$

SECTION- (B)

Q-6 (a) Find the envelope of the ellipse

$$x^2/a^2+y^2/b^2 = 1,$$

where the parameters a and b are connected by $a^2+b^2 = c^2$, c being a constant.

(b) Show that the function f(x,y) is continuous at the origin, where

$$f(x,y) = \begin{cases} (x^2 - y^2)/(x^2 + y^2), & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

Q- 7 (a) By ε- δ technique, prove that the function f: $R^2 \rightarrow R$ defined by

$$f(x,y) = \begin{cases} xy(x^2 - y^2)/(x^2 + y^2), & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

is continuous at (0,0)

(b) If $u = \tan^{-1} xy/\sqrt{(1+x^2+y^2}$, prove that

$$\partial^2 u/\partial x\, \partial y = 1/(1+x^2+y^2)^{3/2}$$

Q- 8 (a) If Pdx + Qdy+ Rdz can be made a perfect differential of some function of x,y,z on multiplication by a factor then prove that

$$P(\partial Q/\partial z - \partial R/\partial y) + Q(\partial R/\partial x - \partial P/\partial z) + R(\partial P/\partial y - \partial Q/\partial x) = 0$$

(b) Discuss the maximum or minimum values of

$$f(x,y) = xy + a^3/x + a^3/y$$

Q-9 (a) Find an evolute of $y^2 = 4ax$

(b) If $y_1 = x_2x_3/x_1$, $y_2 = x_1x_3/x_2$, $y_3 = x_1x_2/x_3$, then prove that

$$\partial(y_1,y_2,y_3)/\partial(x_1,x_2,x_3) = 4.$$